Online Facility Location and Steiner Problems

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Summary

Combinatorial Optimization Problems:

- Facility Location, Steiner Tree, Connected Facility Location.

Online Computation and Competitive Analysis:

- Facility Location family problems,
- Steiner family problems,
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Competitive Analysis of the Online Single-Source Rent-or-Buy algorithm.

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Problems with an objective function to be minimized or maximized.

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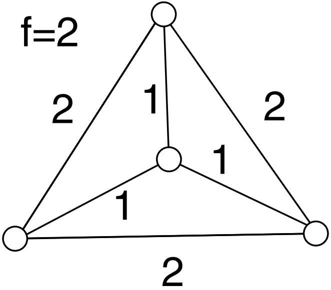
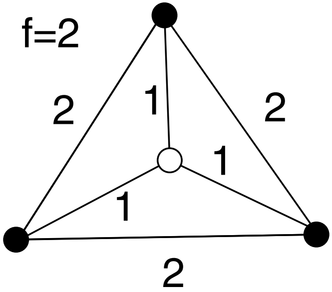


$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a)$$

Total cost $= 2 + 3 = 5$.

Facility Location Problem



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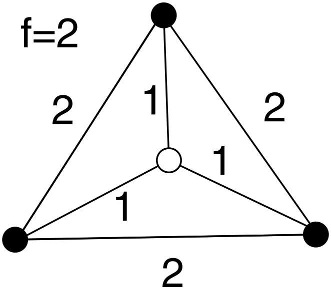

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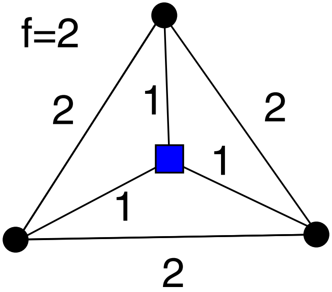

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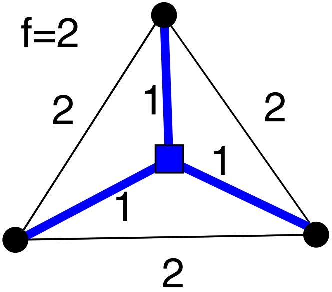

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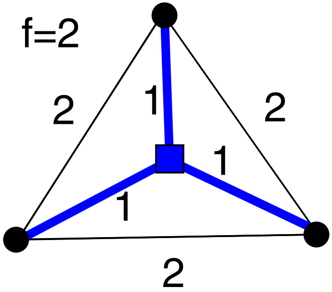

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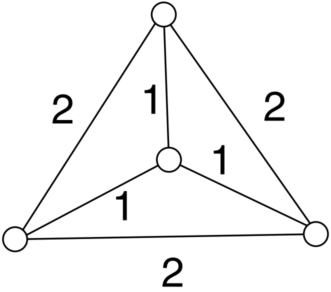
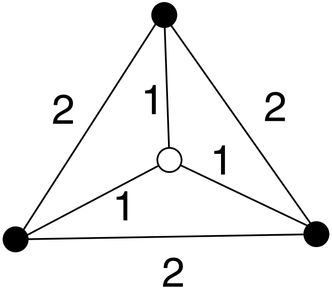


$$\min \sum_{e \in E(T)} d(e)$$

Total cost = 3.

Steiner Tree Problem



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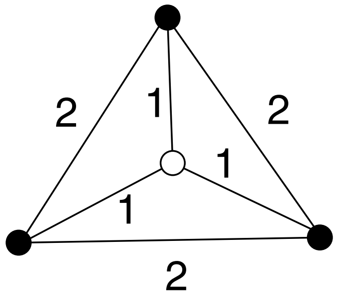

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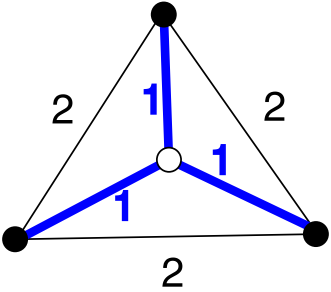

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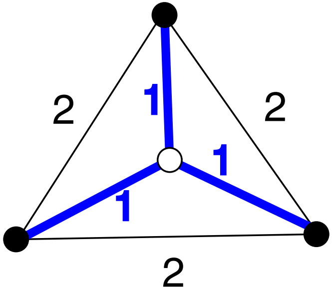

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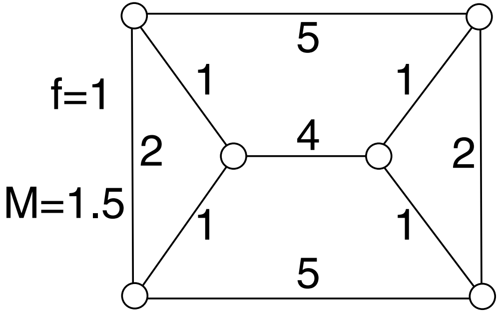
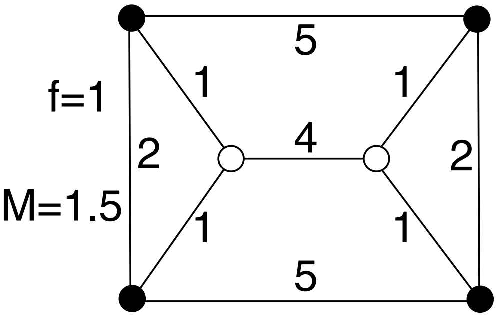


$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a) + M \sum_{e \in E(T)} d(e)$$

Total cost $= 2 + 4 + 6 = 12$.

Connected Facility Location Problem



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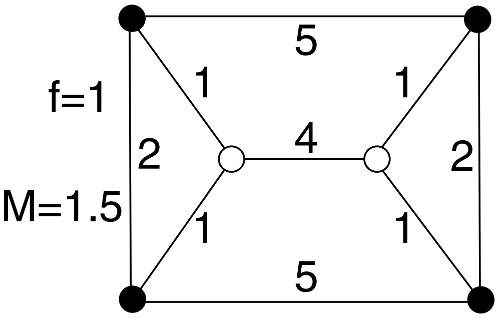

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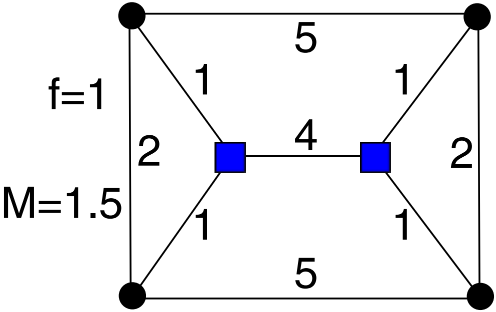

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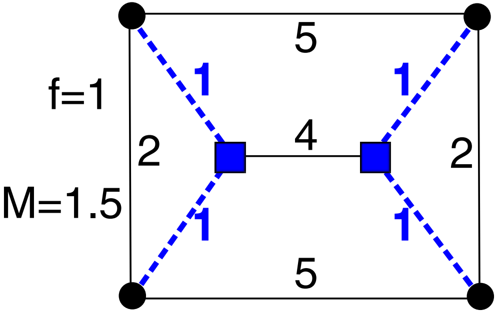

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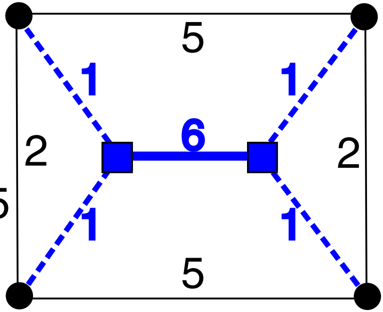

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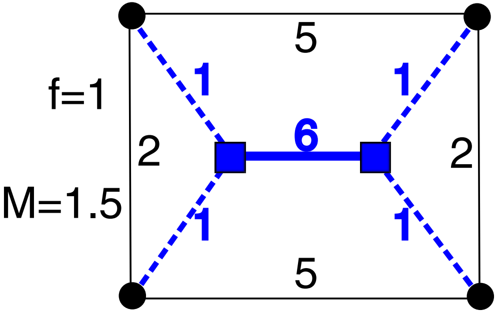

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Parts of the input are revealed one at a time.

Each part must be served before the next one arrives.

No decision can be changed in the future.

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Worst case technique used to analyze online algorithms.

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Minimization problems in which we are interested:

- Online Facility Location (OFL),
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- Online Steiner Tree (OST),
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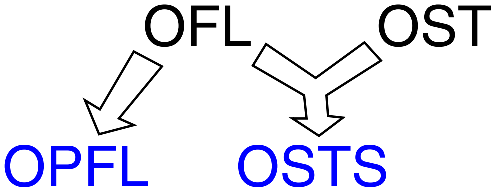

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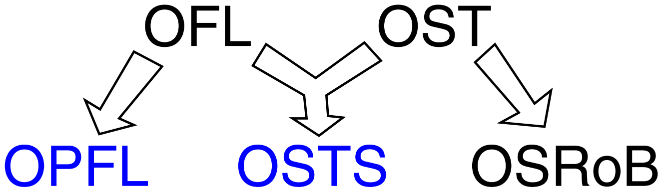

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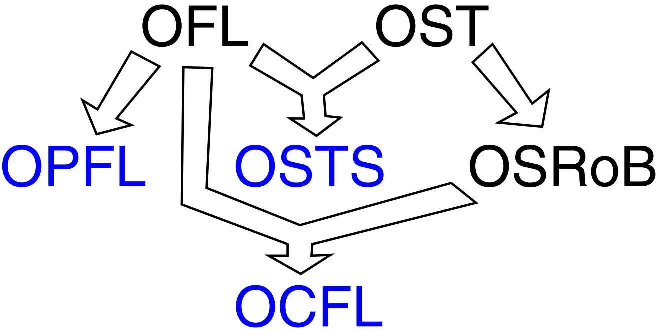

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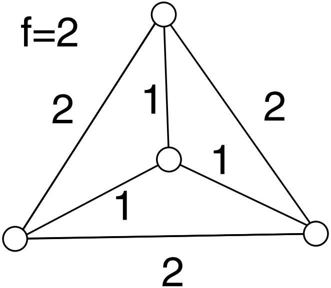

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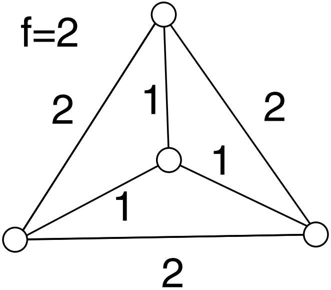

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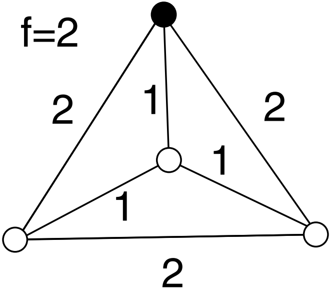

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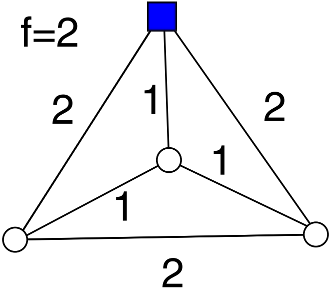

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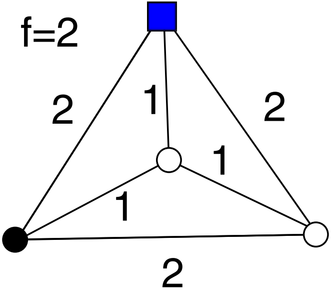

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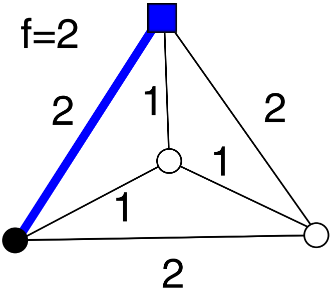

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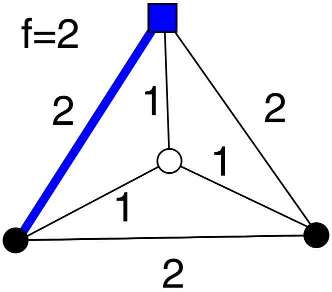

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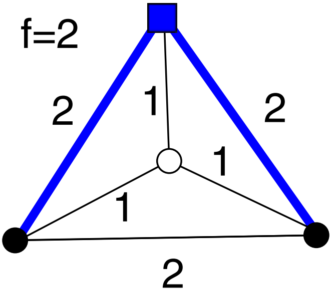

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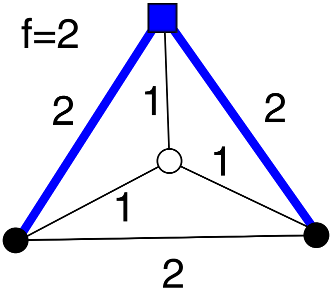

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Online Facility Location Results

There are $O(\log n)$-competitive algorithms known for it.

We show a primal-dual $(4 \log n)$-competitive algorithm by Fotakis [2007] and by Nagarajan and Williamson [2013].

There is a lower bound of $\Omega\left(\frac{\log n}{\log \log n}\right)$ due to Fotakis [2008].

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Online Facility Location LP Formulation

Linear programming relaxation

min	$\sum_{i \in F} f(i) y_i + \sum_{j \in D} \sum_{i \in F} d(j,i) x_{ji}$	
s.t.	$x_{ji} \leq y_i$	for $j \in D$ and $i \in F$,
	$\sum_{i \in F} x_{ji} \geq 1$	for $j \in D$,
	$y_i \geq 0, x_{ji} \geq 0$	for $j \in D$ and $i \in F$,

and its dual

max	$\sum_{j \in D} \alpha_j$	
s.t.	$\sum_{j \in D} (\alpha_j - d(j,i))^+ \leq f(i)$	for $i \in F$,
	$\alpha_j \geq 0$	for $j \in D$.

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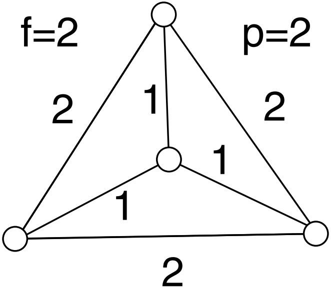
Online Facility Location Algorithm

Algorithm 1: OFL Algorithm.

Input: (G, d, f, F)
$F^a \leftarrow \emptyset$; $D \leftarrow \emptyset$;
while *a new client j' arrives* **do**
    increase $\alpha_{j'}$ until one of the following happens:
    (a) $\alpha_{j'} = d(j', i)$ for some $i \in F^a$; /* connect only */
    (b) $f(i) = (\alpha_{j'} - d(j', i)) + \sum_{j \in D}(d(j, F^a) - d(j, i))^+$ for some $i \in F \setminus F^a$; /* open and connect */
    $F^a \leftarrow F^a \cup \{i\}$; $D \leftarrow D \cup \{j'\}$; $a(j') \leftarrow i$;
end
return (F^a, a);

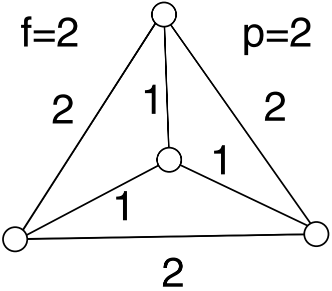
Online Prize-Collecting Facility Location



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D^c} d(j, a(j)) + \sum_{j \in D^p} p(j)$$

Total cost $= 2 + 2 + 2 = 6$.

Online Prize-Collecting Facility Location



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D^c} d(j, a(j)) + \sum_{j \in D^p} p(j)$$

Total cost $= 2 + 2 + 2 = 6$.

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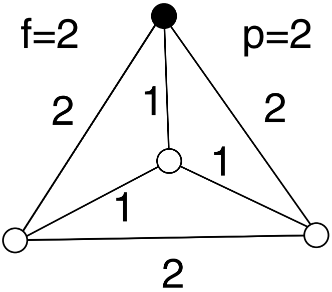

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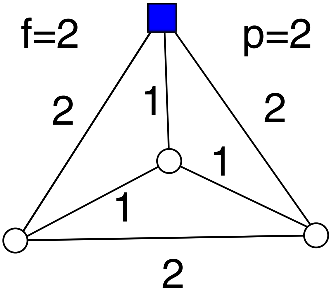

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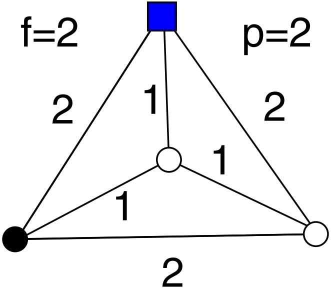

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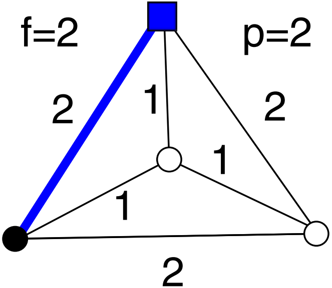

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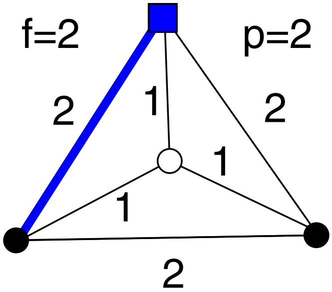

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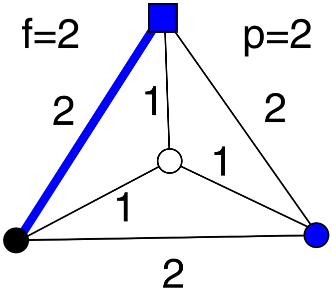

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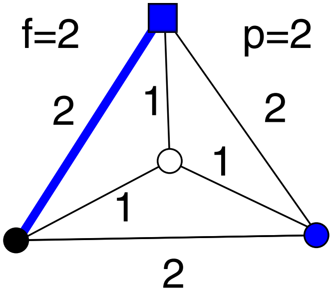

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OPFL Results

We proposed the problem and showed a primal-dual $(6 \log n)$-competitive algorithm for it.

Since it is a generalization of the OFL, the lower bound of $\Omega\left(\frac{\log n}{\log \log n}\right)$ applies to it.

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OPFL LP Formulation

Linear programming relaxation

$$\begin{array}{lll} \min & \sum_{i\in F} f(i)y_i + \sum_{j\in D}\sum_{i\in F} d(j,i)x_{ji} + \sum_{j\in D} p(j)z_j & \\ \text{s.t.} & x_{ji} \le y_i & \text{for } j\in D \text{ and } i\in F, \\ & \sum_{i\in F} x_{ji} + z_j \ge 1 & \text{for } j\in D, \\ & y_i\ge 0, x_{ji}\ge 0, z_j\ge 0 & \text{for } j\in D \text{ and } i\in F, \end{array}$$

and its dual

$$\begin{array}{lll} \max & \sum_{j\in D} \alpha_j & \\ \text{s.t.} & \sum_{j\in D} (\alpha_j - d(j,i))^+ \le f(i) & \text{for } i\in F, \\ & \alpha_j \le p(j) & \text{for } j\in D, \\ & \alpha_j \ge 0 & \text{for } j\in D. \end{array}$$

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s.t.	$x_{ji} \leq y_i$	for $j \in D$ and $i \in F$,
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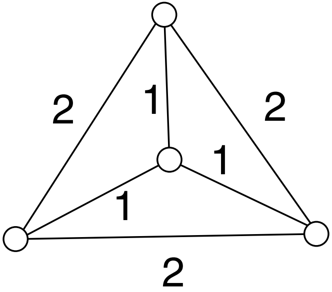
OPFL Algorithm

Algorithm 2: OPFL Algorithm.

Input: (G, d, f, p, F)
$D \leftarrow \emptyset$; $F^a \leftarrow \emptyset$;
while *a new client j' arrives* **do**
  increase $\alpha_{j'}$ until one of the following happens:
  (a) $\alpha_{j'} = d(j', i)$ for some $i \in F^a$; /* connect only */
  (b) $f(i) = (\alpha_{j'} - d(j', i)) + \sum_{j \in D}(\min\{d(j, F^a), p(j)\} - d(j, i))^+$ for some $i \in F \setminus F^a$; /* open and connect */
  (c) $\alpha_{j'} = p(j')$; /* pay the penalty */
  (in this case i is choose to be null, i.e., $\{i\} = \emptyset$)
  $F^a \leftarrow F^a \cup \{i\}$; $D \leftarrow D \cup \{j'\}$; $a(j') \leftarrow i$;
end
return (F^a, a);

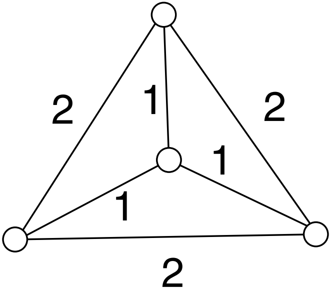
Online Steiner Tree Problem



$$\min \sum_{e \in E(T)} d(e)$$

Total cost $= 2 + 2 = 4$.

Online Steiner Tree Problem



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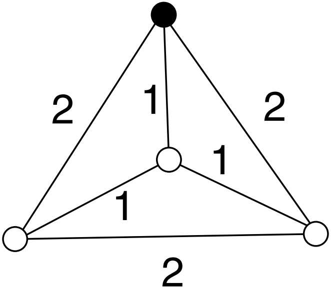

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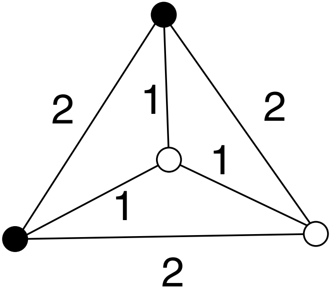

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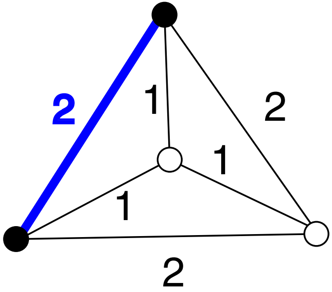

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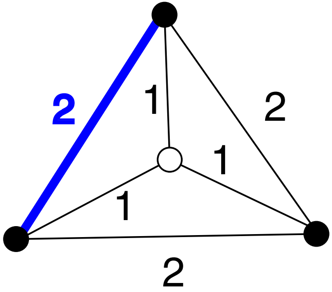

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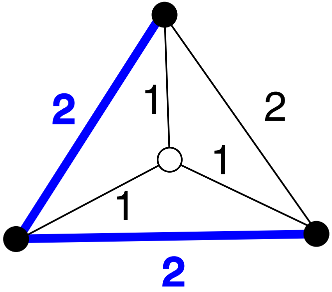

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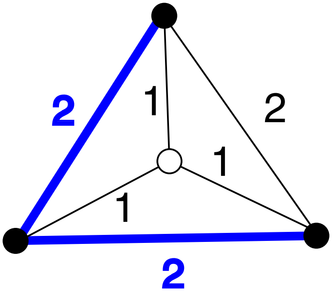

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Online Steiner Tree Results

There are $O(\log n)$-competitive algorithms known for it.

We show a greedy $\lceil \log n \rceil$-competitive algorithm by Imase and Waxman [1991].

There is a lower bound of $\Omega(\log n)$ due to Imase and Waxman [1991].

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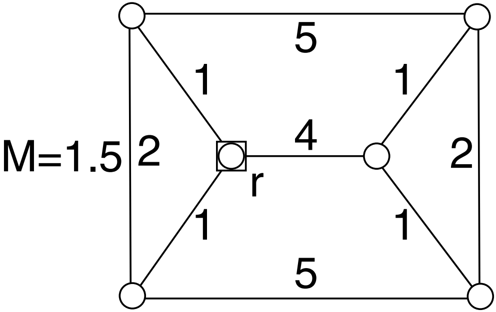
Online Steiner Tree Algorithm

Algorithm 3: OST Algorithm.

Input: (G, d)
$T \leftarrow (\emptyset, \emptyset)$; $D \leftarrow \emptyset$;
while *a new terminal j arrives* **do**
    $T \leftarrow T \cup \{\mathrm{path}(j, V(T))\}$; /* connect */
    $D \leftarrow D \cup \{j\}$;
end
return T;

Online Single-Source Rent-or-Buy Problem



$$\min \sum_{j \in D} \sum_{e \in E(P(j))} d(e) + M \sum_{e \in E(T)} d(e)$$

Total cost $= 1 + 7.5 + 1 + 1 = 10.5$.

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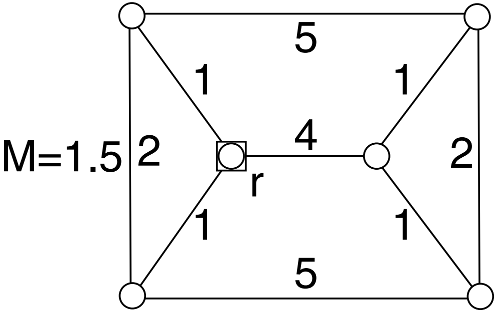

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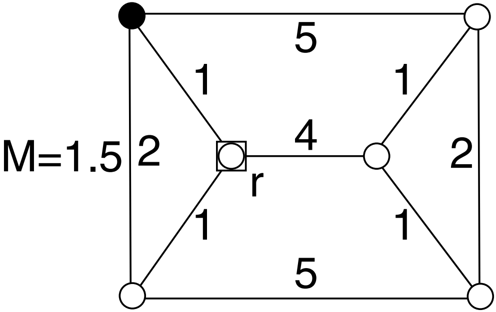

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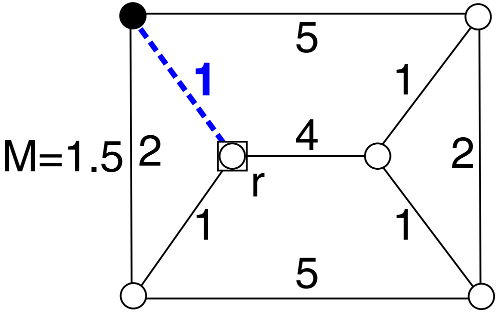

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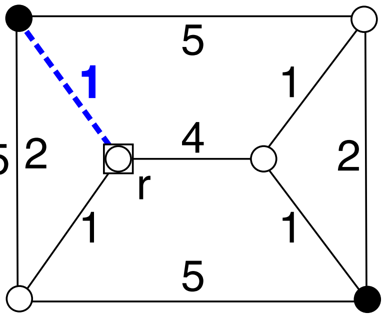

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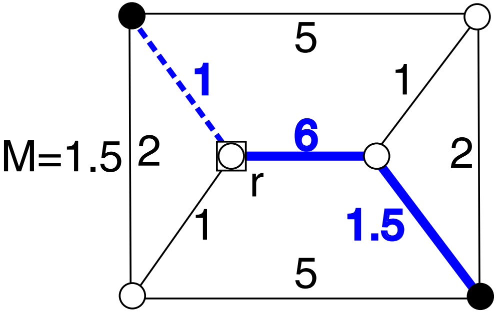

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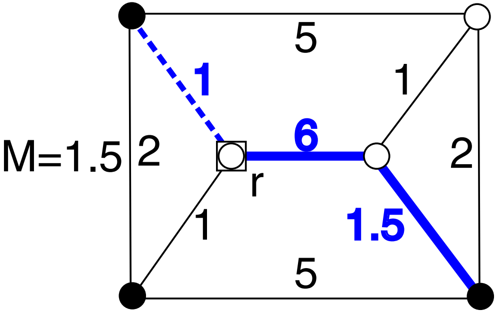

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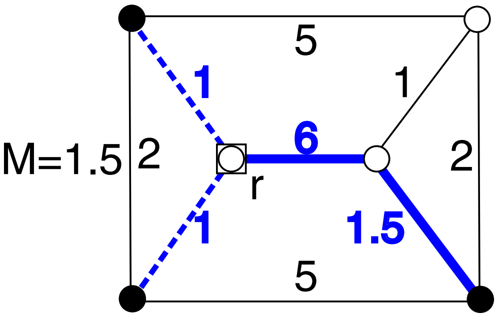

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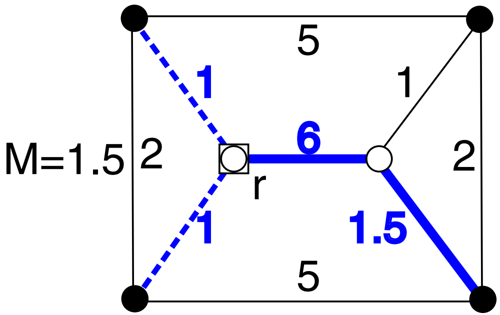

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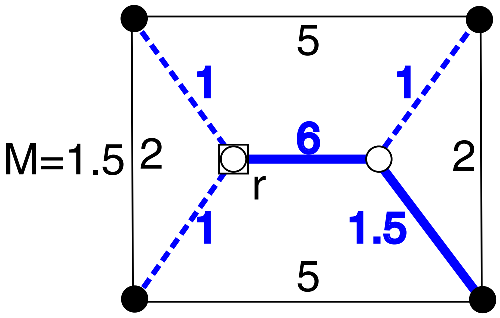

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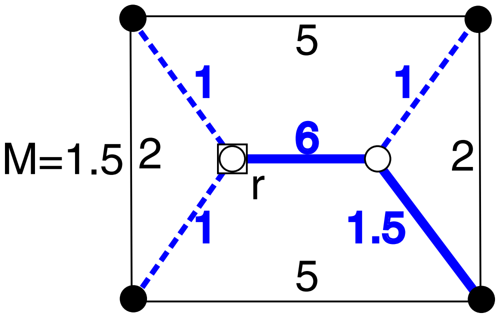

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There is a sample-and-augment $2\lceil \log n \rceil$-competitive algorithm by Awerbuch, Azar and Bartal [2004].

We show that algorithm and a simpler analysis for it.

Since this problem is a generalization of the OST, the lower bound of $\Omega(\log n)$ applies to it.

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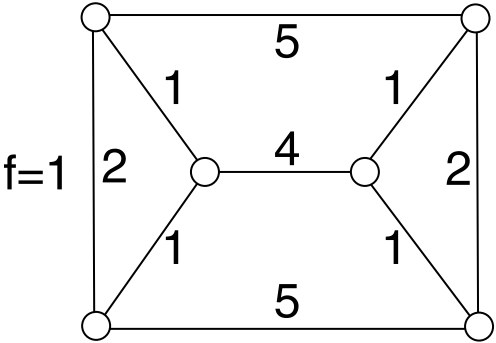
Online Single-Source Rent-or-Buy Algorithm

Algorithm 4: OSRoB Algorithm.

Input: (G, d, r, M)
$T \leftarrow (\{r\}, \emptyset)$; $P \leftarrow \emptyset$; $D \leftarrow \emptyset$; $D^m \leftarrow \emptyset$;
while *a new terminal j arrives* **do**
    include j in D^m with probability $\frac{1}{M}$;
    if $j \in D^m$ **then**
        $T \leftarrow T \cup \{\mathrm{path}(j, V(T))\}$; /* buy edges */
    end
    $P(j) \leftarrow \mathrm{path}(j, V(T))$; /* rent edges */
    $P \leftarrow P \cup \{P(j)\}$;
    $D \leftarrow D \cup \{j\}$;
end
return (P, T);

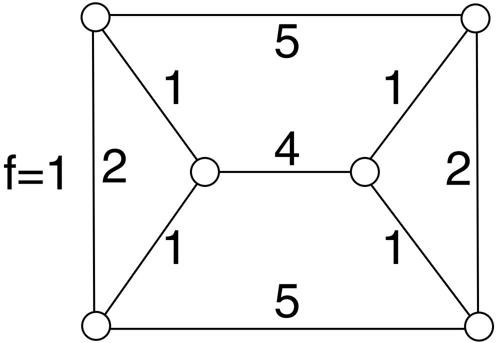
Online Steiner Tree Star Problem



$$\min \sum_{e \in E(T)} d(e) + \sum_{\substack{i \in V(T):\\ \delta_T(i) > 1}} f(i)$$

Total cost $= 1 + 5 + 2 + 1 + 1 = 10.$

Online Steiner Tree Star Problem



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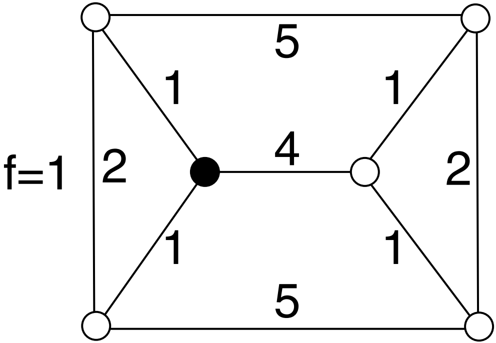

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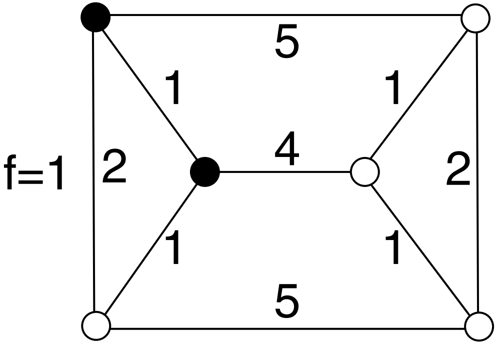

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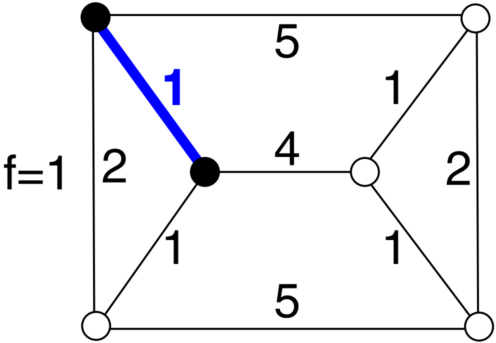

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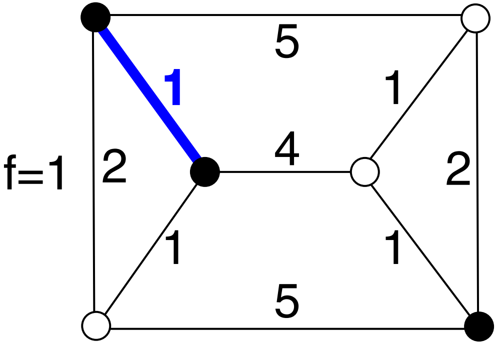

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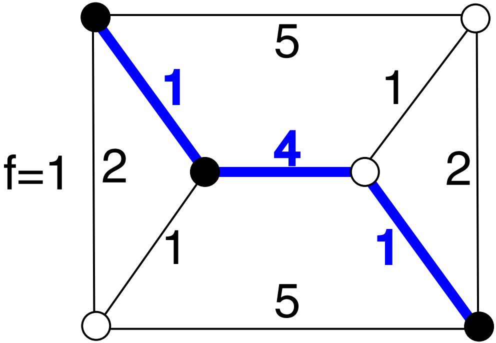

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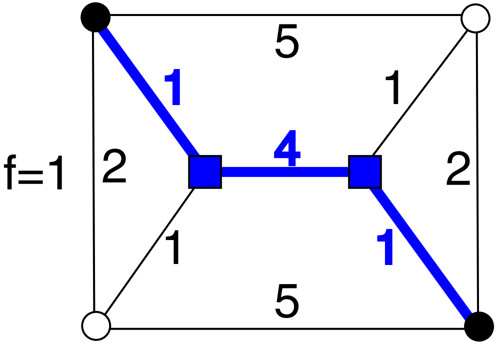
Total cost $= 1 + 5 + 2 + 1 + 1 = 10.$

Online Steiner Tree Star Problem



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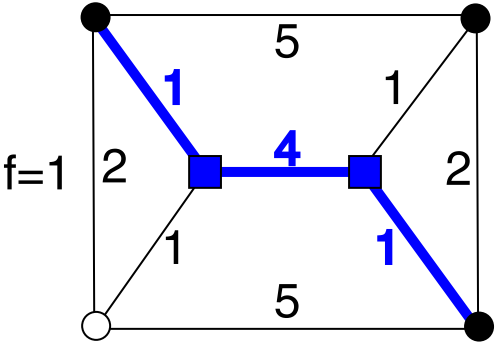

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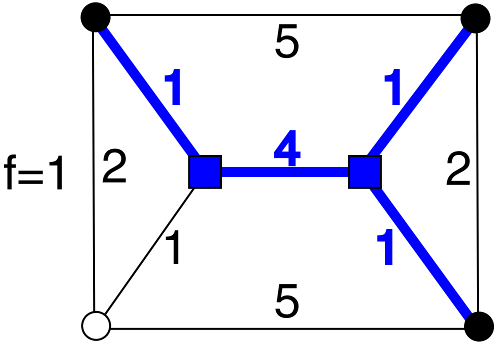

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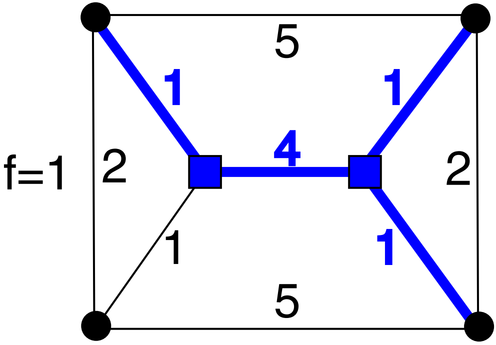

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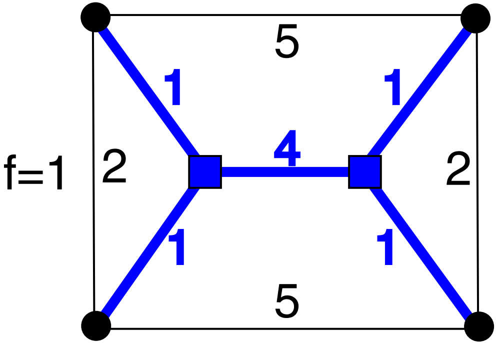

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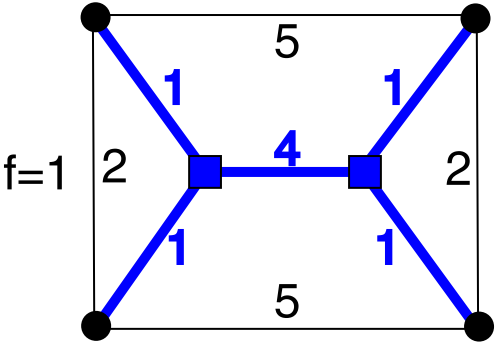

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Online Steiner Tree Star Results

We proposed the problem and showed a $3\lceil \log^2 n \rceil$-competitive algorithm for it, for $n \geq 17$.

Since it is a generalization of the OST, the lower bound of $\Omega(\log n)$ applies to it.

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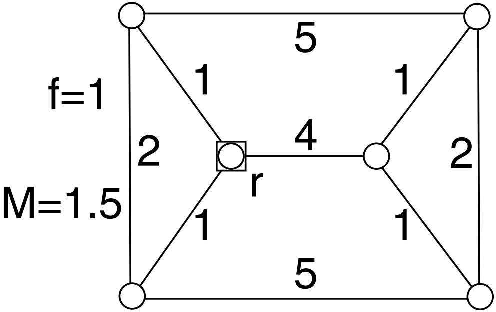
Online Steiner Tree Star Algorithm

Algorithm 5: OSTS Algorithm.

Input: (G, d, f)
initialize $\mathrm{ALG}_{\mathrm{OFL}}$ with (G, d, f, V);
$T^c \leftarrow (\emptyset, \emptyset)$; $T^x \leftarrow (\emptyset, \emptyset)$; $D \leftarrow \emptyset$;
while *a new terminal j arrives* **do**
    send j to $\mathrm{ALG}_{\mathrm{OFL}}$;
    if *$a(j)$ is not in $V(T^c)$* **then**
        $T^c \leftarrow T^c \cup \{\mathrm{edge}(a(j), V(T^c))\}$; /* connect internal node */
    end
    $T^x \leftarrow T^x \cup \{\mathrm{edge}(j, a(j))\}$; /* connect leaf */
    $D \leftarrow D \cup \{j\}$;
end
$T \leftarrow T^c \cup T^x$;
return T;

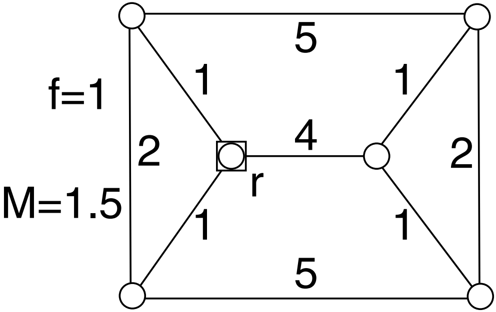
Online Connected Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j)) + M \sum_{e \in E(T)} d(e)$$

Total cost $= 1 + 1 + 7.5 + 1 + 2 = 12.5$.

Online Connected Facility Location Problem



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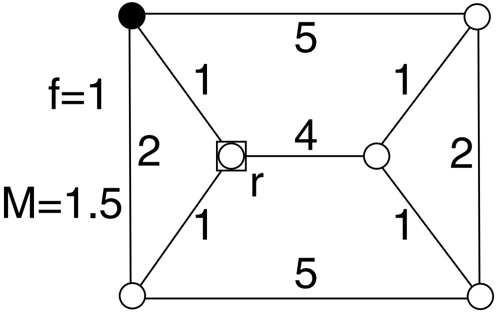

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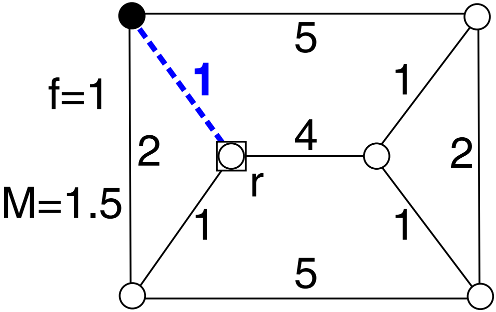

$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j)) + M \sum_{e \in E(T)} d(e)$$

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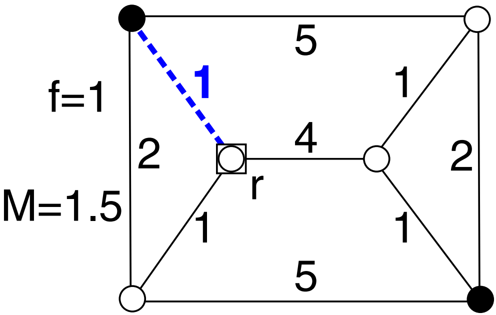
Online Connected Facility Location Problem



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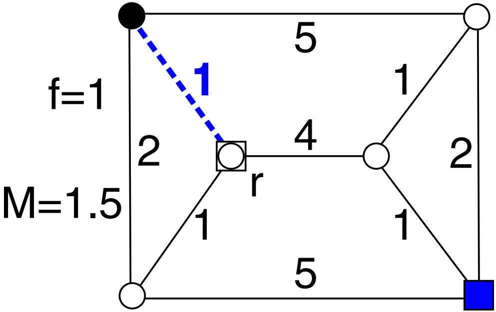

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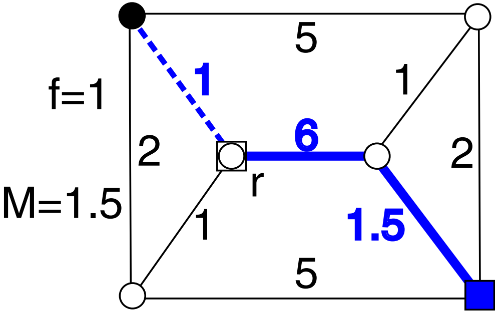

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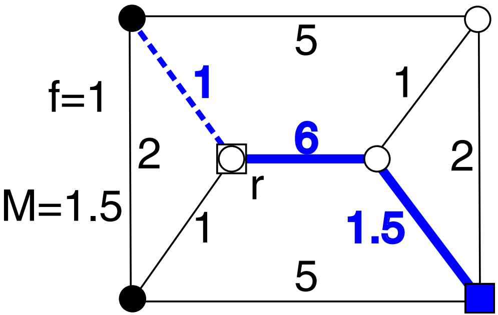

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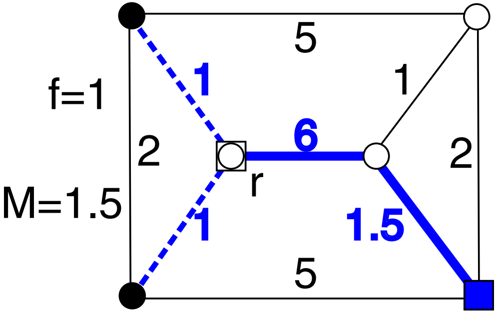

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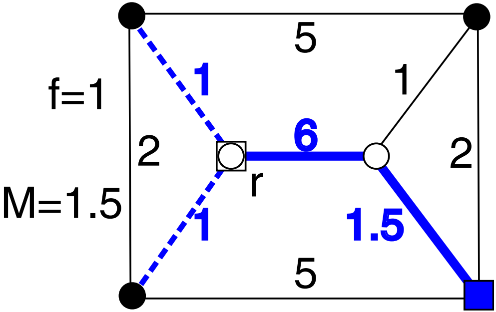

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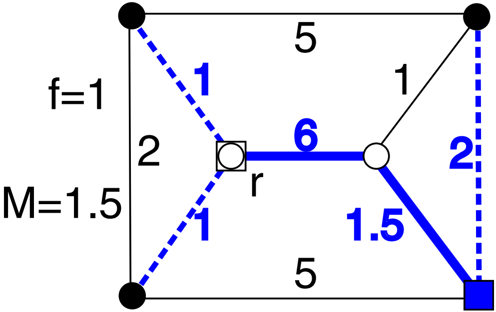

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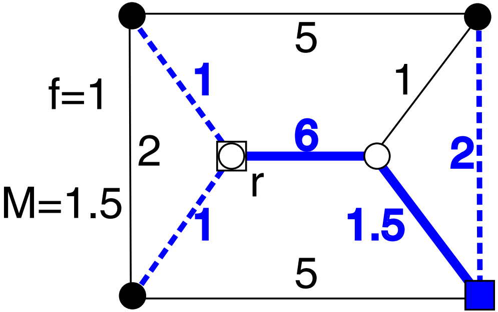

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OCFL Results

We proposed the problem and showed a sample-and-augment $18\lceil \log n \rceil$-competitive algorithm for it.

We also showed that the same algorithm has competitive ratio $7\lceil \log n \rceil$ for the special case in which $M = 1$.

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Since this problem is a generalization of the OST, the lower bound of $\Omega(\log n)$ applies to it.

Algorithm 6: OCFL Algorithm.

Input: (G, d, f, F, r, M)

set $f(r) \leftarrow 0$ and initialize $\mathrm{ALG_{OFL}}$ with (G, d, f, F);

send r to $\mathrm{ALG_{OFL}}$; $F^a \leftarrow \{r\}$; $T \leftarrow (\{r\}, \emptyset)$;

while *a new client j arrives* **do**

  send j to $\mathrm{ALG_{OFL}}$; /* update virtual solution */

  include j in D^m with probability $\frac{1}{M}$;

  if $j \in D^m$ **then**

    $T \leftarrow T \cup \{\mathrm{path}(j, V(T))\}$; /* connect new facility */

    if *$v(j)$ is not opened* **then**

      $F^a \leftarrow F^a \cup \{v(j)\}$; $T \leftarrow T \cup \{(v(j), j)\}$;

    end

  end

  choose $i \in F^a$ that is closest to j; $D \leftarrow D \cup \{j\}$; $a(j) \leftarrow i$;

end

return (F^a, a, T);

Analysis of the OSRoB Algorithm

We are going to show the following result.

Theorem

$$E[\mathrm{ALG}_{\mathrm{OSRoB}}(D_n)] \leq 2\lceil \log n \rceil \mathrm{OPT}_{\mathrm{SRoB}}(D_n) \ .$$

Analysis of the OSRoB Algorithm

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$E[\mathrm{ALG}_{\mathrm{OSRoB}}(D_n)] \leq 2\lceil \log n \rceil \mathrm{OPT}_{\mathrm{SRoB}}(D_n)$.

Analysis of the OSRoB Algorithm

We want to compare

$$\mathrm{ALG}_{\mathrm{OSRoB}}(D_n) = \sum_{j \in D_n} \sum_{e \in E(P_n(j))} d(e) + M \sum_{e \in E(T_n)} d(e) = R(D_n) + B(D_n) \ ,$$

with

$$\mathrm{OPT}_{\mathrm{SRoB}}(D_n) = \sum_{j \in D_n} \sum_{e \in E(P_n^*(j))} d(e) + M \sum_{e \in E(T_n^*)} d(e) = R^*(D_n) + B^*(D_n) \ .$$

Analysis of the OSRoB Algorithm

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Remembering the OSRoB Algorithm

Algorithm 7: OSRoB Algorithm.

Input: (G, d, r, M)
$T \leftarrow (\{r\}, \emptyset)$; $P \leftarrow \emptyset$; $D \leftarrow \emptyset$; $D^m \leftarrow \emptyset$;
while *a new terminal j arrives* **do**
  include j in D^m with probability $\frac{1}{M}$;
  if $j \in D^m$ **then**
    $T \leftarrow T \cup \{\mathrm{path}(j, V(T))\}$; /* buy edges */
  end
  $P(j) \leftarrow \mathrm{path}(j, V(T))$; /* rent edges */
  $P \leftarrow P \cup \{P(j)\}$;
  $D \leftarrow D \cup \{j\}$;
end
return (P, T);

Analysis of the OSRoB Algorithm

Note that

$$R(D_n) = \sum_{j \in D_n} \sum_{e \in E(P_n(j))} d(e)$$

$$= \sum_{j \in D_n \setminus D_n^m} d(j, V(T_{n(j)})) = \sum_{j \in D_n} r(j) \ .$$

Also, note that

$$B(D_n) = M \sum_{e \in E(T_n)} d(e)$$

$$= M \sum_{j \in D_n^m} d(j, V(T_{n(j)-1})) = \sum_{j \in D_n} b(j) \ .$$

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Analysis of the OSRoB Algorithm

Now we bound the expected buying cost.

Lemma

$E[B(D_n)] \leq \lceil \log n \rceil \mathrm{OPT}_{\mathrm{SRoB}}(D_n)$.

Analysis of the OSRoB Algorithm

Demonstração.

$$
\begin{aligned}
E[B(D_n)] = E\left[\sum_{j\in D_n^m} b(j)\right] &\leq ME\left[\sum_{j\in D_n^m} d(j, V(T_{n(j)-1}))\right]\\
&\leq ME[\mathrm{ALG_{OST}}(D_n^m)] \leq M\lceil \log n\rceil E[\mathrm{OPT_{ST}}(D_n^m)]\\
&\leq M\lceil \log n\rceil \left(\frac{B^*(D_n)}{M} + E\left[\sum_{j\in D_n^m} d(j, V(T^*_{n(j)}))\right]\right)\\
&= \lceil \log n\rceil \left(B^*(D_n) + M\sum_{j\in D_n}\frac{d(j, V(T^*_{n(j)}))}{M}\right)\\
&= \lceil \log n\rceil \left(B^*(D_n) + R^*(D_n)\right)\\
&= \lceil \log n\rceil \mathrm{OPT_{SRoB}}(D_n)\ .
\end{aligned}
$$

Analysis of the OSRoB Algorithm

Demonstração.

$$E[B(D_n)] = E\left[\sum_{j\in D_n^m} b(j)\right] \leq ME\left[\sum_{j\in D_n^m} d(j, V(T_{n(j)-1}))\right]$$

$$\leq ME[\mathrm{ALG_{OST}}(D_n^m)] \leq M\lceil\log n\rceil E[\mathrm{OPT_{ST}}(D_n^m)]$$

$$\leq M\lceil\log n\rceil\left(\frac{B^*(D_n)}{M} + E\left[\sum_{j\in D_n^m} d(j, V(T^*_{n(j)}))\right]\right)$$

$$= \lceil\log n\rceil\left(B^*(D_n) + M\sum_{j\in D_n}\frac{d(j, V(T^*_{n(j)}))}{M}\right)$$

$$= \lceil\log n\rceil\left(B^*(D_n) + R^*(D_n)\right)$$

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Analysis of the OSRoB Algorithm

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Analysis of the OSRoB Algorithm

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&= \lceil \log n \rceil \left(B^*(D_n) + M \sum_{j \in D_n} \frac{d(j, V(T^*_{n(j)}))}{M} \right) \\
&= \lceil \log n \rceil \left(B^*(D_n) + R^*(D_n) \right) \\
&= \lceil \log n \rceil \mathrm{OPT}_{\mathrm{SRoB}}(D_n) \ .
\end{aligned}
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Analysis of the OSRoB Algorithm

And now we bound the expected renting cost.

Lemma

$E[R(D_n)] \leq E[B(D_n)]$.

Analysis of the OSRoB Algorithm

Demonstração.

Let $E[x(j)|n(j)-1]$ be the random variable $x(j)$ conditioned to the first $n(j)-1$ random choices of the algorithm. Thus

$$\begin{aligned} E[r(j)|n(j)-1] &= \frac{M-1}{M} d(j, V(T_{n(j)})) \\ &\leq d(j, V(T_{n(j)-1})) \\ &= \frac{1}{M} M d(j, V(T_{n(j)-1})) \leq E[b(j)|n(j)-1] \ . \end{aligned}$$

Since this holds for any outcome of the first $n(j)-1$ random choices of the algorithm, it holds unconditionally. So

$$E[R(D_n)] = \sum_{j \in D_n} E[r(j)] \leq \sum_{j \in D_n} E[b(j)] = E[B(D_n)] \ .$$

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Analysis of the OSRoB Algorithm

Demonstração.

Using the two previous lemmas we have that:

$$\begin{aligned} E[\mathrm{ALG}_{\mathrm{OSRoB}}(D_n)] &\leq E[R(D_n)] + E[B(D_n)] \\ &\leq 2\lceil \log n \rceil \mathrm{OPT}_{\mathrm{SRoB}}(D_n) \quad . \end{aligned}$$

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Conclusions

The main results:

- a primal-dual $(6 \log n)$-competitive algorithm for the OPFL, to appear on LAGOS 2015;
- a $3\lceil \log^2 n \rceil$-competitive algorithm for the OSTS, for $n \geq 17$;
- a sample-and-augment $18\lceil \log n \rceil$-competitive algorithm for the OCFL (and deterministic $7\lceil \log n \rceil$-competitive algorithm for the special case in which $M = 1$).
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